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Refining and Expanding the Geometry Pedagogical Improvement Cycle (GeoPIC): A Conceptual Contribution to Competency-Based Geometry Instruction

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Abstract: *This conceptual paper refines and expands the Geometry Pedagogical Improvement Cycle (GeoPIC), a framework designed to improve the teaching and learning of geometry. Rooted in the Van Hiele Theory of Geometric Thinking, GeoPIC was initially developed to integrate the strengths of both Conventional Van Hiele Phased Instruction and its technology-enhanced variant while addressing their contextual limitations. Building on prior empirical findings, the paper deepens the framework's theoretical foundations, clarifies its six instructional phases, and illustrates how it aligns with the principles of Competency-Based Curriculum (CBC). Additionally, it offers brief, practical classroom scenarios to illustrate the application of each phase. The refined framework provides a dynamic, adaptive, and learner-centered instructional approach that facilitates progression through Van Hiele levels while accommodating diverse classroom contexts.*

Keywords*: Van Hiele Theory, Competency-Based Curriculum, Technology-Enhanced Learning, GeoGebra, Conceptual Understanding, Constructivist Learning, GeoPIC Framework.*

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1. Introduction

Despite the structured nature of geometry curricula and the adoption of proven pedagogical models, such as the Van Hiele Theory of Geometric Reasoning, many learners continue to struggle with conceptual understanding in geometry, particularly in transformation geometry (Dahal et al., 2022; Malatjie & Machaba, 2019; Ndungo, Balimuttajjo, & Akugizibwe, 2025a; Ngwabe & Felix, 2020). The Van Hiele model, which outlines a progression of five reasoning levels, has been widely applied to guide instructional strategies in this domain. However, while the theory provides a solid foundation, the practical implementation of its phases often faces challenges, especially in diverse classroom contexts.

Conventional Van Hiele Phased Instruction (CVHPI) promotes procedural fluency through the use of structured, hands-on tools. In contrast, Technology-Enhanced Van Hiele Phased Instruction (TVHPI) leverages dynamic visualization via digital tools, such as GeoGebra.

However, both approaches face context-specific limitations, underscoring the need for an adaptive framework (Ndungo, 2024; Ndungo et al., 2024).

These pedagogical tensions, between traditional and digital, between structure and adaptability, highlight the need for a more integrative instructional model. One that not only honors the sequential logic of Van Hiele's theory but also adapts to learners' diverse needs, supports reflective teaching, and aligns with Competency-Based Curriculum (CBC) principles. Such a model must be empirically grounded to ensure relevance and effectiveness in real classroom environments. It is from this need that Ndungo and colleagues proposed the Geometry Pedagogical Improvement Cycle (GeoPIC). The development of this framework was rooted in a rigorous empirical study aimed at understanding how instructional strategies influence learners' conceptual progression in transformation geometry (Ndungo, Balimuttajjo, & Akugizibwe, 2025b).

Although the Geometry Pedagogical Improvement Cycle (GeoPIC) was proposed as a flexible framework for enhancing instruction in transformation geometry, its full potential lies in how it responds to the real-world complexities of classroom implementation. Specifically, GeoPIC was developed to navigate the instructional tensions that emerge when applying Van Hiele-based strategies in diverse school contexts, especially those characterized by disparities in technological access, teacher preparedness, and learner engagement.

At its core, GeoPIC acknowledges that Technology-Enhanced Van Hiele Phased Instruction (TVHPI) is not a standalone method but an enrichment of the Conventional Van Hiele Phased Instruction (CVHPI). TVHPI builds on CVHPI's structured pedagogical phases by incorporating dynamic digital tools such as GeoGebra, thus enhancing visualization and interactivity. However, classroom evidence shows that the advantages of TVHPI are not evenly distributed (Mukamba & Makamure, 2020). In well-resourced urban settings, learners benefit from dynamic and explorative features. At the same time, in rural or low-tech environments, these enhancements can become constraints due to limited access, erratic power supply, or insufficient training (Ndungo, Balimuttajjo, & Akugizibwe, 2025a, 2025b).

Moreover, learner characteristics further complicate instructional decisions. For instance, while some students thrive in tech-rich environments that promote autonomy and exploration, others, particularly those with low initial confidence or minimal prior exposure, respond better to hands-on, step-by-step guidance offered through CVHPI (Kay, 2006). This diversity necessitates a responsive, adaptive instructional model rather than a fixed approach.

GeoPIC was therefore conceptualized not simply to merge TVHPI and CVHPI, but to provide a cyclical framework through which teachers can make informed, context-sensitive decisions. It positions instructional strategy selection as part of a broader pedagogical process, one that begins with diagnosing learner needs, aligning expectations, responding to challenges, and continually refining the instructional approach.

In Uganda's CBC context, this flexibility is particularly crucial. The CBC mandates the integration of ICT, learner-centered approaches, and formative assessment, yet it also operates within a landscape marked by infrastructural disparities (NCDC, 2020). GeoPIC offers a way to translate the CBC's ambitions into practical, classroom-level decisions, supporting both conceptual rigor and instructional realism.

Ultimately, the rationale for GeoPIC lies in its ability to adapt instruction to the varied and shifting conditions of real classrooms, ensuring that no learner is left behind due to rigid methodology or inaccessible tools. By guiding teachers through a process of planning, reflection, and adaptation, it facilitates more equitable and effective progression through the Van Hiele levels of geometric reasoning.

The purpose of this paper is to elaborate, clarify, and extend the Geometry Pedagogical Improvement Cycle (GeoPIC), a research-derived instructional framework originally formulated from empirical findings comparing Conventional and Technology-Enhanced Van Hiele Phased Instruction. While GeoPIC was initially introduced as a theoretical model for adaptive geometry teaching, this paper provides a detailed explanation of its phases, illustrates its alignment with Competency-Based Curriculum (CBC) principles, and positions it as a flexible, learner-centered guide for improving conceptual understanding in transformation geometry across diverse instructional contexts.

2. Literature Review

The Geometry Pedagogical Improvement Cycle (GeoPIC) is grounded in a robust empirical study that explored how different instructional strategies influenced students' experiences and understanding of transformation geometry (Ndungo, 2024; Ndungo et al., 2024; Ndungo, Balimuttajjo, & Akugizibwe, 2025a, 2025b; Ndungo, Nazziwa, et al., 2025). The study was conducted in Ugandan secondary schools and examined the effectiveness of two approaches: Conventional Van Hiele Phased Instruction (CVHPI) and its Technology-Enhanced counterpart (TVHPI), which incorporates tools like GeoGebra. Through a mixed-methods design involving pre-tests, post-tests, and in-depth interviews with learners, the study revealed that while TVHPI significantly improved learners' attitudes and engagement, particularly in urban settings, it also posed challenges related to infrastructure and dropout sensitivity. Conversely, CVHPI demonstrated stability and consistency, especially in low-resource contexts, but sometimes lacked the interactive richness needed to foster deeper conceptual understanding (Ndungo, Balimuttajjo, & Akugizibwe, 2025b; Ndungo, Balimuttajjo, Akugizibwe, et al., 2025; Ndungo, Balimuttajjo, & Akugizibwe, 2025c)

These findings highlighted the need for a more flexible and adaptive instructional model, one that can integrate the strengths of both CVHPI and TVHPI while responding to contextual challenges, such as learner diversity, resource availability, and curriculum demands. In response to this need, the GeoPIC framework was proposed as a cyclical, research-informed model to guide geometry instruction.

At the core of the GeoPIC framework lies the Van Hiele Theory, which outlines five hierarchical levels of geometric reasoning: (1) Visualization, (2) Analysis, (3) Informal Deduction, (4) Deduction, and (5) Rigor. According to the theory, learners must master one level before making meaningful progress to the next, and this advancement is not age-dependent but rather instructional-method-dependent (Crowley, 1987; Fuys et al., 1988; Usiskin, 1982; Van Hiele, 1957; Vojkuvkova, 2012).

The comparative study that gave rise to the GeoPIC framework demonstrated how both CVHPI (Conventional Van Hiele Phased Instruction) and TVHPI (Technology-Enhanced VHPI using GeoGebra) were aligned with this model. However, CVHPI often emphasized hands-on tools and procedural guidance, which helped many low-achieving learners remain grounded in their cognitive development. In contrast, TVHPI's use of dynamic visualizations enabled faster conceptual shifts in learners already operating at higher Van Hiele levels. However, it posed risks of cognitive overload or shallow engagement when learners "moved shapes" without reasoning through the transformations.

GeoPIC acknowledges these instructional trade-offs. It structures learning experiences into cyclical phases that facilitate not only level progression but also support re-entry into earlier stages if misconceptions are identified, thus aligning deeply with the non-linear but hierarchical nature of Van Hiele's framework.

Informed by the work of Piaget and Vygotsky, constructivist learning theory emphasizes that learners build understanding through active engagement, exploration, and reflection, rather than passively receiving facts (Alanazi, 2016; Allen, 2022; Banihashem et al., 2022; Gatt & Vela, 2003; Mcleod, 2023; Narayan et al., 2013; Tobias, 2010; Yilmaz, 2008). GeoPIC operationalizes this by integrating self-explanation, diagnostic assessment, and guided practice within each instructional phase.

During the fieldwork underpinning GeoPIC, students exposed to TVHPI described how exploring transformations through GeoGebra helped them "see what the shapes were doing" and "test what happens if I rotate more." However, others noted they only understood the underlying principles after drawing the same transformation manually. This aligns with constructivist tenets, which hold that manipulation alone is insufficient unless paired with reflection and articulation. By cycling between interactive experiences and reflection, GeoPIC fosters metacognitive development, enabling students to monitor and regulate their learning, a key constructivist goal (Narayan et al., 2013).

Another key theoretical underpinning of GeoPIC is Differentiated Instruction, which asserts that effective teaching must account for learners' varying readiness levels, interests, and learning profiles (Lavania & Nor, 2020). The original study revealed stark contrasts between learners in urban versus rural settings. Urban students, familiar with digital tools, showed enthusiasm for GeoGebra-based tasks. In contrast, rural students often expressed a preference for traditional hands-on tasks that used rulers, mirrors, and graph paper.

GeoPIC addresses this by not prescribing a single mode of instruction but offering an adaptive structure. Teachers are encouraged to assess the instructional context and learner characteristics to determine whether to begin with CVHPI, TVHPI, or a hybrid approach. This responsiveness ensures inclusivity and effectiveness across diverse classrooms, particularly in contexts such as Uganda's CBC rollout, where learner variability is substantial.

GeoPIC's cyclical nature draws inspiration from the Plan-Do-Check-Act (PDCA) model, originally conceptualized by Walter A. Shewhart and later popularized by W. Edwards Deming for process improvement, which is increasingly applied in education (Pratik & Vivek, 2017). Each phase of GeoPIC represents an actionable stage: instructional planning and adoption (Plan), learner engagement and teaching (Do), diagnosis and reflection (Check), and strategy refinement (Act). Rather than viewing instruction as a linear sequence of lessons, GeoPIC frames it as an iterative loop where feedback informs constant recalibration.

This principle was mirrored in classrooms, where teachers, after observing persistent learner difficulties with concepts such as inverse transformations or angle measurement, paused the instructional sequence, revisited earlier phases with scaffolded tasks, and reinforced learning through both manual and digital tools. In doing so, they reinforced the idea that teaching must be responsive, not rigid.

3. Methodology

While this paper is conceptual, it draws from prior empirical research aimed at evaluating the effectiveness of two instructional strategies: Conventional Van Hiele Phased Instruction (CVHPI) and its Technology-Enhanced variant (TVHPI). The study was grounded in the Van Hiele Theory of Geometric Reasoning. The study involved 483 students from six secondary schools in Uganda, stratified by gender, academic achievement level, and school location (urban or rural).

The intervention design implemented both instructional strategies across different groups, with TVHPI utilizing digital tools, primarily GeoGebra, and CVHPI relying on physical teaching aids, such as graph paper and tracing materials. Data collection included pre- and post-tests, classroom observations, and semi-structured interviews. The qualitative component focused on learners' experiences, their perceived instructional effectiveness, and the challenges they encountered. Thematic analysis revealed a pattern of phased learning experiences that guided learners from visual recognition to abstract reasoning, closely aligned with the Van Hiele levels of understanding.

These findings led to the formulation of the GeoPIC framework, which integrates practical features of both instructional strategies into a dynamic cycle designed to support conceptual progression, metacognitive reflection, and continuous instructional improvement. The original version of GeoPIC was proposed to explain how adaptive, feedback-driven instruction can better support geometry learning within the Competency-Based Curriculum (CBC) context in Uganda.

This paper builds upon the original empirical study by offering a conceptual expansion and clarification of the GeoPIC framework. It aims to articulate the theoretical and instructional rationale behind the model, present its phases in detail, and illustrate its application in modern classroom contexts. This transformation from research output to a conceptual framework required a systematic synthesis of prior data, literature, and reflective analysis.

For this conceptual paper, the author revisited the findings of the original study, identifying the pedagogical logic behind each phase of the GeoPIC cycle. This was followed by an extensive narrative literature review that brought together constructs from the Van Hiele Theory, constructivist learning models, reflective teaching practices, and differentiated instruction.

Next, each phase of the cycle was elaborated using illustrative examples, educational rationale, and connections to the original data. To enhance clarity, a new visual representation of the framework was created to reflect the iterative and cyclical nature of GeoPIC. The revised visual and narrative model captures both the teacher's and learner's roles within each phase, offering a practical guide for implementation and further research.

In this way, the current work was developed through a combination of empirical grounding, theoretical refinement, and pedagogical translation. It does not report new field data, but instead interprets and builds upon existing research to offer a more precise and richer conceptual understanding of the framework. The outcome is a scholarly and practice-oriented model that is both responsive to classroom realities and grounded in educational theories.

4. Results and Discussion

The core outcome of this paper is the systematic clarification and elaboration of the Geometry Pedagogical Improvement Cycle (GeoPIC). This framework integrates empirical evidence, theoretical constructs, and pedagogical insights into a coherent instructional model for teaching transformation geometry.

The original study that inspired GeoPIC revealed that learners benefit most when instruction is adaptive, phased, and responsive to their conceptual readiness. While CVHPI ensured a structured path for progression through the Van Hiele levels, TVHPI enriched learners' engagement through visual and interactive experiences using GeoGebra. However, limitations in both approaches, such as dropout sensitivity in TVHPI and resource constraints in CVHPI, indicated the need for a hybrid, flexible, and teacher-guided framework (Ndungo, Balimuttajjo, & Akugizibwe, 2025b). This paper extends empirical insight by unpacking the six distinct phases of the GeoPIC framework.

The GeoPIC framework is structured around six interrelated phases that form a continuous cycle of instructional planning, implementation, and reflection. Each phase is intentionally designed to support learners' progression through the Van Hiele levels of geometric reasoning while remaining adaptable to different classroom environments, learner profiles, and curricular expectations. Figure 1 illustrates the six phases of the framework, followed by a detailed explanation for each phase.

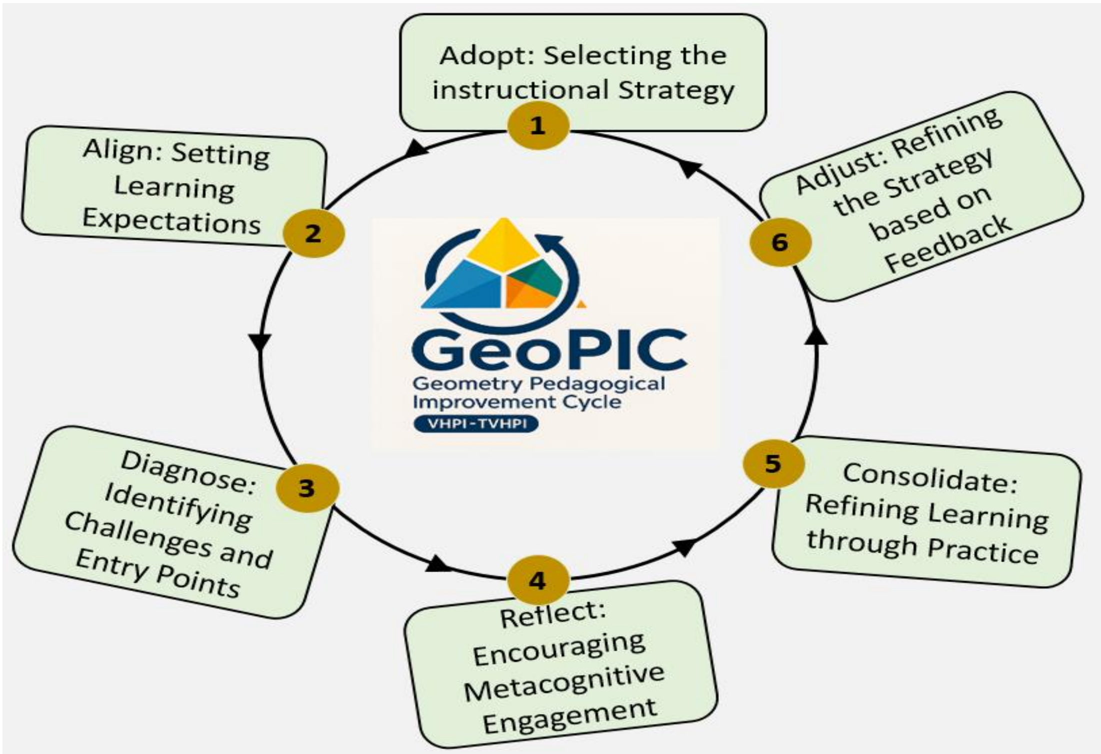

Figure 1: Showing the phases of the Geometry Pedagogical Improvement Cycle (GeoPIC-Framework)

1. Adopt: Selecting the Instructional Strategy

This phase focuses on instructional decision-making. The teacher determines whether to apply Conventional Van Hiele Phased Instruction (CVHPI), which emphasizes hands-on activities using tools like graph paper, mirrors, and rulers, or to adopt Technology-Enhanced Van Hiele Phased Instruction (TVHPI), which integrates digital platforms such as GeoGebra for dynamic visualization. The selection is informed by learners' prior experiences, resource availability, and the complexity of the transformation geometry concepts to be addressed. While not part of the Van Hiele phases themselves, this strategic adoption ensures that instruction begins from an informed and purposeful foundation.

2. Align: Setting Learning Expectations

Once the strategy is selected, the teacher communicates clear learning goals and assessment criteria. This phase aligns with the 'Information' phase of Van Hiele, where learners are introduced to the topic and begin forming connections (Vojkuvkova, 2012). The GeoPIC framework emphasizes transparency and learner ownership at this stage. Students are not only informed about what they will learn, but also how they will engage with the material and how their understanding will be evaluated. In TVHPI contexts, this might include introducing students to the GeoGebra interface and setting up expectations for independent exploration.

3. Diagnose: Identifying Challenges and Entry Points

Here, teachers conduct diagnostic assessments through observation, formative pre-tests, and discussions to identify learners' misconceptions and determine their current Van Hiele level. This stage builds upon the Van Hiele model by explicitly acknowledging that learners enter instruction at different stages of reasoning. Rather than assuming uniform readiness, GeoPIC equips teachers to tailor content and support based on real-time understanding of learners' needs. In doing so, it reinforces the Directed Orientation phase by shaping the experiential activities to match learner readiness (Crowley, 1987; Larbi, 2021; Naufal et al., 2021).

4. Reflect: Encouraging Metacognitive Engagement

In this phase, learners engage in self-reflection, peer discussions, and explanation of thought processes, often through journaling, guided questions, or debriefing sessions after a GeoGebra activity. Teachers also reflect on instructional choices and student responses, identifying what worked and what needs revision. This mirrors and enriches the Explication phase in Van Hiele's model by fostering metacognition and reinforcing the use of geometric language, both of which are crucial for developing deductive reasoning (Moru et al., 2021; Vojkuvkova, 2012).

5. Consolidate: Reinforcing Learning through Practice

This phase corresponds to the Free Orientation and Integration phases of Van Hiele, during which learners begin to solve more complex tasks and consolidate their understanding of the material. Under GeoPIC, consolidation is structured through varied application tasks, paper-based constructions, digital manipulations, real-life modeling, and assessments. This stage is crucial for learners to apply transformations across various contexts and to demonstrate a transition from procedural execution to conceptual understanding (Eisenhart et al., 1993; Mills, 2016; Ndungo, Balimuttajjo, & Akugizibwe, 2025a).

6. Adjust: Refining the Strategy Based on Feedback

Teachers continue improving instruction even after consolidation. In this final phase, teachers analyze learning outcomes and adjust strategies for subsequent cycles. This stage is not explicitly addressed in Van Hiele's original model but is vital in today's competency-based and differentiated instruction contexts. Adjustments may involve re-grouping learners, revisiting challenging concepts, or integrating new digital tools. It parallels the continuous improvement loop found in professional development models such as Plan-Do-Check-Act (Pratik & Vivek, 2017).

Classroom Implementation of the GeoPIC Framework

In a typical geometry classroom, the teacher aims to introduce the concept of reflection across the y-axis. Intending to enable learners to determine the image of a shape after reflection, the teacher begins with the adopt phase, where a decision is made between using conventional instructional methods and enhancing them with technology. Considering the learners' context, the teacher adopts a blended approach that combines traditional tools, such as graph paper and mirrors, with digital platforms like GeoGebra, making the learning process both tangible and dynamic.

Next, the align phase is initiated as the teacher communicates the expected learning outcomes. Learners should be able to draw a shape, reflect it across the y-axis, and accurately label the new coordinates. By making these expectations explicit, learners are mentally prepared and guided towards specific competencies.

As the lesson unfolds, the diagnosis phase becomes essential. The teacher assigns a simple task: reflect a triangle across the y-axis. While monitoring student work, it becomes clear that some students confuse the y-axis with the x-axis. This insight enables the teacher to identify learning gaps and provide targeted support where needed. In this phase, understanding the learners' entry points helps refine the instructional path.

The class then moves into the reflection phase, where learners are encouraged to reflect on their understanding. Students are asked to explain the reasoning behind their reflections and how they know their results are correct. Whether using a mirror or testing their answers in GeoGebra, this moment promotes metacognitive engagement, allowing learners to internalize the concept more deeply.

With clarity starting to emerge, the consolidation phase follows. Learners are given more examples to practice reflecting on different shapes and to compare solutions with peers. This repeated engagement reinforces learning, solidifies understanding, and builds learner confidence.

Finally, the teacher enters the adjustment phase by reviewing students' work and reflecting on the overall effectiveness of the lesson. Observing that many learners struggle with shapes located far from the axis, the teacher decides to incorporate more real-life scenarios and scaffolds in future lessons. This adjustment ensures that future instruction is informed by evidence from classroom practice.

Through this smooth and thoughtful cycle, the Geometry Pedagogical Improvement Cycle (GeoPIC) empowers both the teacher and learners. It offers a practical and adaptable framework that supports progression through the Van Hiele levels, while addressing the real challenges and opportunities present in diverse classroom settings.

Understanding the distinct responsibilities and expectations placed upon both the teacher and the learner is crucial for effectively implementing the GeoPIC framework. Each of the six phases in the Geometry Pedagogical Improvement Cycle (GeoPIC) represents a strategic step that requires active participation and thoughtful engagement from both parties. The teacher serves not merely as a facilitator of content but as a responsive guide who adapts strategies based on learner feedback. In contrast, the learner is expected to transition from a passive reception of knowledge to an active construction of knowledge. These evolving roles reflect the dynamic and cyclical nature of the framework, where teaching and learning are continuously refined. Table 1 below summarizes the defining characteristics of each phase and highlights the corresponding roles played by the teacher and the learner, offering a practical lens through which the phases can be interpreted and applied in real classroom settings.

Table 1: Phase characteristics and roles of the teacher and the learner in the GeoPIC Framework

Phase	Characteristics	Teacher's Role	Learner's Role
1. Adopt	Selects appropriate instructional tools (CVHPI or TVHPI) based on learner needs and context.	Analyze learner profiles and choose between traditional and technology-enhanced strategies.	Get ready to use learning tools like graph paper and GeoGebra.
2. Align	Clarifies learning goals, processes, and assessment methods.	Communicate objectives, set a pacing strategy, and design an engagement path.	Understand learning trajectory, expectations, and how success will be measured.
3. Diagnose	Identifies learners' misconceptions, entry points, and readiness levels.	Use pre-tests, discussions, and observations to detect gaps and strengths.	Participate actively, express understanding, and discuss difficulties.
4. Reflect	Encourages self-reflection and teacher analysis of instructional effectiveness.	Facilitate metacognitive prompts and adapt teaching strategies in response to feedback.	Reflect on learning experiences, assess one's thinking, and articulate reasoning.
5. Consolidate	Provides structured practice through real-world tasks and dynamic exploration.	Design applied tasks using manipulatives or digital tools; offer timely feedback.	Apply concepts in various contexts, test conjectures, and seek a deeper understanding.
6. Adjust	Continuously revises instructions to match learner progress and feedback.	Modify pace, tools, and methods in response to evolving learner needs and feedback to ensure optimal learning outcomes.	Adapt to new strategies, consolidate knowledge, and continually refine your approach to improve.

By detailing each of these phases, the paper demonstrates how GeoPIC moves beyond a linear application of Van Hiele's phases into a reflective, recursive model. This cyclical nature ensures that instruction remains responsive and data-informed, meeting learners at their level of readiness and supporting continuous improvement.

Furthermore, this framework is highly compatible with Competency-Based Curriculum (CBC) principles, which emphasize learner agency, skill acquisition, and contextualized knowledge. GeoPIC aligns with these goals by ensuring instruction is not only conceptually sequenced but also inclusive, differentiated, and driven by classroom realities. The revised visual diagram also enhances usability, helping teachers and researchers alike to conceptualize instructional flow, learner progression, and opportunities for feedback and recalibration.

In summary, the elaborated GeoPIC model serves not only as a product of research but also as a tool for ongoing instructional innovation, encouraging educators to balance structured guidance with adaptive facilitation. It bridges theory and practice, offering a roadmap for meaningful progression in geometric reasoning.

5. Conclusion, Recommendations, and Limitations of the Framework

5.1 Conclusion

This paper aims to clarify, elaborate, and present the Geometry Pedagogical Improvement Cycle (GeoPIC) in a structured manner, a framework grounded in the Van Hiele Theory of Geometric Thinking and developed through the reflective interpretation of empirical data. By systematically detailing its six recursive phases, the paper demonstrates how GeoPIC addresses the instructional gaps inherent in both Conventional Van Hiele Phased Instruction (CVHPI) and its technology-enhanced variant (TVHPI).

The resulting framework supports a holistic, learner-centered approach to transformation geometry by integrating expectations alignment, diagnostic teaching, metacognitive reflection, application-based learning, and iterative instructional adjustment. Moreover, its design resonates with the demands of Competency-Based Curriculum (CBC) implementation, making it a timely and relevant model for contemporary mathematics education in Uganda and similar contexts.

5.2 Recommendations

Recommendations for Educational Practice

Educators should adopt the GeoPIC framework as a planning and reflection tool in teaching geometry. Teachers should move beyond a procedural emphasis to a cycle of diagnostic, reflective, and application-based instruction that ensures learners make meaningful progress through Van Hiele levels.

Recommendations for Policy

Curriculum developers and education policymakers should consider integrating the principles of GeoPIC into national mathematics teacher training programs, CBC instructional guides, and in-service professional development. This would strengthen teachers' capacity to deliver differentiated and theory-driven geometry instruction.

Recommendations for Research

Further research should explore the effectiveness and adaptability of the GeoPIC framework using rigorous designs such as cluster-randomized trials and design-based implementation research. Cross-country comparative studies are also recommended to assess its applicability across different educational contexts.

5.3 Limitations of the Framework

The GeoPIC framework is highly valuable for enhancing geometry teaching, but its effective implementation requires adequate resources, teacher training, and sufficient instructional time. In contexts with limited resources or exam-driven curricula, full adoption can be challenging, and ongoing support is essential to maximize its impact.

References

Alanazi, A. (2016). A Critical Review of Constructivist Theory and the Emergence of Constructionism. *American Research Journal of Humanities and Social Sciences*, *2*, 1–8. https://doi.org/10.21694/2378-7031.16018

Allen, A. (2022). An Introduction to Constructivism : Its Theoretical Roots and Impact on Contemporary Education. *Journal of Learning Design and Leadership*, *1*(1), 1–11. https://doi.org/https://doi.org/

Banihashem, S. K., Farrokhnia, M., Badali, M., & Noroozi, O. (2022). The impacts of constructivist learning design and learning analytics on students' engagement and self-regulation. *Innovations in Education and Teaching International*, *59*(4), 442–452. https://doi.org/10.1080/14703297.2021.1890634

Crowley, M. (1987). The van Hiele model of the development of geometric thought. *Learning and Teaching Geometry, K-12*, 1–16. http://www.csmate.colostate.edu/docs/math/mathactivities/june2007/The

Dahal, N., Pant, B. P., Shrestha, I. M., & Manandhar, N. K. (2022). Use of GeoGebra in Teaching and Learning Geometric Transformation in School Mathematics. *International Journal of Interactive Mobile Technologies*, *16*(8), 65–78. https://doi.org/10.3991/ijim.v16i08.29575

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual Knowledge Falls through the Cracks: Complexities of Learning to Teach Mathematics for Understanding. *Journal for Research in Mathematics Education*, *24*(1), 8. https://doi.org/10.2307/749384

Fuys, D., Geddes, D., & Tischler, R. (1988). The Van Hiele Model of Thinking in Geometry among Adolescents. *Journal for Research in Mathematics Education. Monograph*, *3*, i--196. http://www.jstor.org/stable/749957

Gatt, S., & Vela, Y. (2003). Constructivism: An Effective Theory of Learning. *Constructivist Teaching in Primary Schools: Examples in Social Studies, Science, Mathematics, Design and Technology and ICT*, *November*, 1–20. https://www.researchgate.net/publication/284722693

Kay, R. (2006). Addressing gender differences in computer ability, attitudes and use: The laptop effect. *Journal of Educational Computing Research*, *34*(2), 187–211. https://doi.org/10.2190/9BLQ-883Y-XQMA-FCAH

Larbi, E. (2021). Assessing van Hiele's geometric thinking levels among elementary pre-service mathematics teachers. *African Educational Research Journal*, *9*(4), 844–851. https://doi.org/10.30918/aerj.94.21.119

Lavania, M., & Nor, F. B. M. (2020). Barriers in differentiated instruction: A systematic review of the literature. *Journal of Critical Reviews*, *7*(6), 293–297. https://doi.org/10.31838/jcr.07.06.51

Malatjie, F., & Machaba, F. (2019). Exploring mathematics learners' conceptual understanding of coordinates and transformation geometry through concept mapping. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12), 1–16. https://doi.org/10.29333/EJMSTE/110784

Mcleod, S. (2023). Constructivism Learning Theory & Philosophy of Education. *Www.Simplypsychology.Org*, 1–14. https://www.simplypsychology.org/constructivism.html#Constructivism-Philosophy

Mills, S. (2016). Conceptual understanding: A concept analysis. *Qualitative Report*, *21*(3), 546–557. https://doi.org/10.46743/2160-3715/2016.2308

Moru, E. K., Malebanye, M., Morobe, N., & George, M. J. (2021). A Van Hiele Theory analysis for teaching the volume of three-dimensional geometric shapes. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *6*(1), 17–31. https://doi.org/10.23917/jramathedu.v6i1.11744

Mukamba, E., & Makamure, C. (2020). Integration of GeoGebra in Teaching and Learning Geometric Transformations at Ordinary Level in Zimbabwe. *Contemporary Mathematics and Science Education*, *1*(1), ep20001. https://doi.org/10.30935/conmaths/8431

Narayan, R., Rodriguez, C., Araujo, J., Shaqlaih, A., & Moss, G. (2013). Constructivism—Constructivist learning theory. In *The handbook of educational theories.* (pp. 169–183). IAP Information Age Publishing. https://psycnet.apa.org/record/2013-11995-015

Naufal, M. A., Abdullah, A. H., Osman, S., Abu, M. S., Ihsan, H., & Rondiyah. (2021). Reviewing the Van Hiele model and the application of metacognition on geometric thinking. *International Journal of Evaluation and Research in Education*, *10*(2), 597–605. https://doi.org/10.11591/ijere.v10i2.21185

NCDC. (2020). *Lower Secondary Curriculum*. National Curriculum Development Centre, Uganda. https://www.ncdc.go.ug/images/DOCS/resources/Mathematics_Syllabus_compressed.pdf

Ndungo, I. (2024). An Assessment of the Alignment of Transformation Geometry with Van Hiele Levels in Uganda's Lower Secondary Mathematics Curriculum. *Turkish Journal of Mathematics Education*, *5*(2), 33–41. https://tujme.org/index.php/tujme/article/view/108

Ndungo, I., Akugizibwe, E., & Balimuttajjo, S. (2024). Analyzing trends and suggested instructional strategies for Geometry education : Insights from Uganda Certificate of Education-Mathematics Examinations , 2016-2022. *African Journal of Teacher Education*, *13*(2), 153–186. https://journal.lib.uoguelph.ca/index.php/ajote/article/view/8106

Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025a). Conceptual and Procedural Trajectories in Transformation Geometry: A Cmparative study of Technology-enhanced and Conventional Van Hiele Phased Instruction. *International Jouranal of Education Technology and Science*, *5*(2), 160–184. https://doi.org/10.5281/zenodo.15499128

Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025b). Towards Improved Geometry Instruction : Learners ' Experiences with Technology-Enhanced and Conventional Van Hiele Phased Instruction. *The Electronic Journal for Research in Science & Mathematics Education*, *29*(2), 33–56. https://doi.org/https://ejrsme.icrsme.com/article/view/23815

Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025c). Transformation Geometry Attitude Scale (TGAS): Development and Validation for Secondary School Learners in Uganda. *International Journal of Advanced Research*, *8*(1), 140–150. https://doi.org/10.37284/ijar.8.1.2861

Ndungo, I., Balimuttajjo, S., Akugizibwe, E., & Muwonge, M. C. (2025). Modeling Learners ' Attitudinal Shifts in Transformation Geometry under Technology-enhanced and Conventional Van Hiele Phased Instruction. *African Journal of Research in Mathematics, Science and Technology Education*, 1–18. https://doi.org/10.1080/18117295.2025.2529124

Ndungo, I., Nazziwa, C., Kabiswa, W., Opio, P., Akugizibwe, E., & Mpungu, K. (2025). Enhancing STEM Instruction through Indigenous Materials and ICT Integration : A Critical Assessment of Teachers ' Knowledge, Experiences , Attitudes , and Readiness. *East African Journal of Education Studies*, *8*(2), 132–149. https://doi.org/10.37284/eajes.8.2.2872

Ngwabe, A., & Felix, C. (2020). Using GeoGebra to Address Students' Misconceptions about the Transformation of Algebraic Hyperbola Functions. *African Journal of Research in Mathematics, Science and Technology Education*, *24*(3), 348–360. https://doi.org/10.1080/18117295.2020.1854494

Pratik, M. P., & Vivek, A. D. (2017). Application Of Plan-Do-Check-Act Cycle For Quality And Productivity Improvement - A Review. *International Journal for Research in Applied Science & Engineering Technology*, *5*(I), 197–201. https://www.researchgate.net/publication/318743952_Application_Of_Plan-Do-Check-Act_Cycle_For_Quality_And_Productivity_Improvement-A_Review

Tobias, S. (2010). Generative learning theory, paradigm shifts, and constructivism in educational psychology: A tribute to Merl Wittrock. *Educational Psychologist*, *45*(1), 51–54. https://doi.org/10.1080/00461520903433612

Usiskin, Z. (1982). *Van Hiele Levels and Achievement in Secondary School Geometry: Final report of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project* (p. 227).https://eric.ed.gov/?id=ED220288

Van Hiele, P. M. (1957). *The Child's Thought and Geometry*. https://doi.org/https://eric.ed.gov/?id=ED287697

Vojkuvkova, I. (2012). The van Hiele Model of Geometric Thinking. *WDS'12 Proceedings of Contributed Papers*, *1*, 72–75. https://doi.org/https://physics.mff.cuni.cz/wds/proc/pdf12/WDS12_112_m8_Vojkuvkova.pdf

Yilmaz, K. (2008). Constructivism: Its Theoretical Underpinnings, Variations, and Implications for Classroom Instruction. *Educatio Nal Horizons*, *86*(3), 161–172. https://www.jstor.org/stable/42923724